Dynamic Forecasting Rules and the Complexity of Exchange Rate Dynamics[1]

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Abstract

This paper investigates the exchange rate dynamics implied by a heterogeneous agent model proposed in De Grauwe and Grimaldi (2006). The two groups of agents, chartists and fundamentalists, use simple forecasting rules and the ex post relative profitability to decide whether to switch to the other group. We extend this model by introducing a simple evolutionary selection mechanism which allows agents not only to switch between groups but also to adapt the forecasting rule of each group over time. This selection process naturally leads agents to choose forecasting rules over time which results in the convergence of the exchange rate to its fundamental value. However, our learning rule is not robust to the introduction of shocks to the fundamental. In particular, once we allow for random variation in the fundamental, the model exhibits again all of the nonlinear features discussed in De Grauwe and Grimaldi (2006): the disconnect puzzle, volatility clustering and fat tails.

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1. INTRODUCTION

Over the last decades, a growing number of "behavioral" models have been proposed in economics and especially in finance. These behavioral models, implicitly or explicitly, invoke the existence of behavioral biases to argue against the rational expectations, representative agent framework, underlying most standard finance models. Critics of the latter approach have pointed at both the empirical relevance of cognitive biases or bounded rationality as well as the failure of standard models to account for (explain) various stylized facts of asset price dynamics (e.g. bubbles, fat tails, volatility clustering); see, for instance, De Grauwe and Grimaldi (2005a) and Brock and Hommes (1998).

The proposed new behavioral finance paradigm (see, for instance, De Grauwe and Grimaldi (2006)) offers a different view on market dynamics in which groups of boundedly rational agents interact in the economy. These agents use simple forecasting models (rules of thumb) and their limited information prevents them from adapting their beliefs/models according to a (rational) learning rule. Restrictions on the learning capacity of agents prevent the convergence to a full rational expectations equilibrium. As a result, the asset price dynamics implied by behavioral finance models typically show a complex behavior, with substantial and persistent deviations from its fundamental value, fueled by the respective groups (sentiments) in the market.

In this paper, we investigate if standard behavioral finance models are robust to the introduction of an evolutionary selection mechanism, mimicking simple forms of learning. Rather than assuming a set of fixed and non-adaptive forecasting rules, as is common in standard behavioral finance models, we extend the models by allowing for a Darwinian, evolutionary, selection mechanism. Specifically, we allow for new forecasting rules (species) to appear randomly and to be tested by the market (nature). To the extent that such rules outperform existing forecasting rules, agents switch to the new type of model. Unfit forecasting models disappear while new (fitter) models take over. The main question of this paper is whether or not such trial and error processes and survival of the fittest rules could eventually restore the rational expectations benchmark.

We study this question within the general framework of behavioral finance, foreign exchange rate models. The model is a modified version of De Grauwe and Grimaldi (2006).[6] A common assumption in these models is the presence of two types of agents in the market, chartists and fundamentalists.[7] Chartists or technical traders are assumed to use extrapolative trading rules based on past exchange rate movements.[8] Fundamentalists, on

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[6] This model has been recently estimated for the European Monetary System (EMS) period by de Jong et al. (2010).

[7] See Frankel and Froot (1986) for a seminal application of this approach to the exchange rate market and Brock and Hommes (1997, 1998), Lux (1998) and Amilon (2008), among others, for applications to the stock market.

[8] The pervasive use of chartist rules is reported, for example, in Taylor and Allen (1992).

the other hand, believe the exchange rate reverts to its fundamental value at a certain speed. We extend the standard approach. We allow agents to evaluate within each set of forecasting models, i.e. fundamentalist and chartist, different varieties of models (differentiated by either the extrapolative parameter or the mean reversion) and within each class to select (switch to) the most fit rule. New varieties of chartist and fundamentalist forecasting rules appear continuously over time and are tested against the existing rules. The natural selection process of forecasting rules is then followed by assuming that agents choose the better forecasting rule within each class. We use a numerical analysis to study this process of natural selection of forecasting rules and implied exchange rate dynamics. Our numerical analysis shows that this simple mechanism does not fully eliminate the complex price dynamics. Hence, behavioral finance models of the type introduced by De Grauwe and Grimaldi (2006) seem to be robust to the random natural selection processes, which confirms the relevance of behavioral models.

The remainder of the paper is organized as follows. Section 2 presents the exchange rate model proposed by De Grauwe and Grimaldi (2006) and discusses in detail the selection mechanism adopted by both chartists and fundamentalists. Section 3 reports the results from the numerical simulation of the model. Section 4 concludes the paper.

2. THE MODEL

In this section, we first present the heterogeneous agent model for the exchange rate proposed by De Grauwe and Grimaldi (2006) with minor modifications to the forecasting rules used by each group of agents. We then introduce an extension to the model (natural selection) where each group of agents is allowed to modify its forecasting rule over time.

2.1 HETEROGENEOUS AGENT MODEL WITH STATIC FORECASTING RULES

The baseline model assumes the existence of two types of agents, *chartists* (*c*) and *fundamentalists* (*f*), with different expectations regarding the exchange rate. Chartists forecast future exchange rates by simply extrapolating past exchange rate movements. Their forecasting rule is therefore given by:

$$E_t^c(\Delta S_{t+1}) = \beta \sum_{i=1}^{T} \alpha_i \Delta S_{t-i}, \tag{1}$$

where E_t^c represents the expectation operator of the chartist agent, S_t is the exchange rate at time *t*, expressed as the amount of domestic currency per unit of foreign currency, $\sum_{i=1}^{T} \alpha_i = 1$, and β denotes the degree of extrapolation.[9]

[9] As we show below, the market clearing exchange rate at time t depends on forecasts of S_{t+1}. It is assumed that when these forecasts are made S_t has not yet been observed. De Grauwe and Grimaldi (2006) use in its

Fundamentalists believe the exchange rate will return to its fundamental value and therefore use the following, mean reverting, forecasting rule:

$$E_t^f(\Delta S_{t+1}) = -\psi(S_{t-1} - S_{t-1}^*), \tag{2}$$

where ψ denotes the speed of adjustment of the exchange rate towards its fundamental value ($0 < \psi < 1$).[10] Fundamentalists also take transaction costs in the goods market into account. They believe arbitrage in the goods market will drive the exchange rate back to its fundamental value but only when deviations of the exchange rate from the fundamental value are larger than the transaction cost (C) in the goods market:

$$\begin{cases} |S_{t-1} - S_{t-1}^*| > C & \text{Eq. (2) applies,} \\ |S_{t-1} - S_{t-1}^*| < C & E_t^f(\Delta S_{t+1}) = 0. \end{cases} \tag{3}$$

Next to providing a conditional mean, the chartists and fundamentalist forecasting rules also imply a measure of risk, obtained by comparing actual realized exchange rate changes to the (model-specific) forecasts. Denoting the risk measures for the chartist and fundamentalist beliefs by $\sigma_{c,t}^2$ and $\sigma_{f,t}^2$ respectively, we follow De Grauwe and Grimaldi (2006) and posit the following measures (for a full justification of the volatility forecasting rules, see De Grauwe and Grimaldi (2005a,b and 2006):

$$\sigma_{c,t}^2 = (1-\theta)\sigma_{c,t-1}^2 + \theta[E_{t-2}^c(S_{t-1}) - S_{t-1}]^2, \tag{4}$$

$$\sigma_{f,t}^2 = (1-\theta)\sigma_{f,t-1}^2 + \frac{\theta\left[E_{t-2}^f(S_{t-1}) - S_{t-1}\right]^2}{1 + (S_{t-1} - S_{t-1}^*)^2}. \tag{5}$$

We assume that agents are characterized by bounded rationality. Specifically, given expectations, agents maximize expected utility under the standard budget constraint. The standard demand function obtains, relating the excess net demand for foreign currency ($d_{i,t},\ i = c, f$) to the expectations and risk assessment by each type of agent:

$$d_{i,t} = \frac{(1+r^*)E_t^i S_{t+1} - (1+r)S_t}{\mu_i \sigma_{i,t}^2}, \tag{6}$$

where r and r^* denote the domestic and foreign interest rate respectively and μ denotes the risk aversion parameter.

Finally, the relative strengths of the two types of agents determine the final market clearing equilibrium exchange rate. Denoting the fraction of agents of type i ($i = c, f$) by $w_{i,t}$, De Grauwe and Grimaldi (2006) show that the equilibrium exchange rate is

place the previous exchange rate, S_{t-1}. We opt for using the forecasting rule used by the chartist, which gives $E_t^c(S_t) = S_{t-1} + \beta \sum_{i=1}^T \alpha_i \Delta S_{t-i-1}$.

[10] As in the case of the chartists, we assume that fundamentalists use the following rule for the exchange rate at time *t*: $E_t^f(S_t) = S_{t-1} - \psi(S_{t-1} - S_{t-1}^*)$.

determined as a (risk-) weighted average of the specific expectations of the different groups:

$$S_t = \left(\frac{1+r^*}{1+r}\right)\left[\frac{\frac{w_{c,t}}{\sigma^2_{c,t}}}{\frac{w_{c,t}}{\sigma^2_{c,t}}+\frac{w_{f,t}}{\sigma^2_{f,t}}}E^c_t S_{t+1} + \frac{\frac{w_{f,t}}{\sigma^2_{c,t}}}{\frac{w_{c,t}}{\sigma^2_{c,t}}+\frac{w_{f,t}}{\sigma^2_{f,t}}}E^f_t S_{t+1}\right]. \qquad (7)$$

The relative sizes of the respective groups are determined by the relative (past) profitability of the respective forecasting rules. This mechanism was originally proposed by Brock and Hommes (1997) and assumes that weights of specific rules increase with their (recent) profitability:

$$w_{c,t} = \frac{\exp\left[\gamma\pi'_{c,t}\right]}{\exp\left[\gamma\pi'_{c,t}\right]+\exp\left[\gamma\pi'_{f,t}\right]}, \qquad (8)$$

$$w_{f,t} = \frac{\exp\left[\gamma\pi'_{f,t}\right]}{\exp\left[\gamma\pi'_{c,t}\right]+\exp\left[\gamma\pi'_{f,t}\right]}, \qquad (9)$$

where $\pi'_{c,t}$ and $\pi'_{f,t}$ are the risk adjusted returns computed by chartists and fundamentalists, and γ represents the intensity with which agents revise their forecasting rules. The dependence of the weight of the groups on past profitability creates a positive feedback loop which generates significant nonlinearities in the model: increasing profitability of a rule makes the rule more attractive and increases the number of agents using it which subsequently moves the actual market exchange rate more in line with the implied forecasts and profitability. These feedback loops generate nonlinear bubble-like dynamics, reinforcing specific rules until exogenous random shocks reverse the cycle. This mechanism underlies much of the recent literature in behavioral models for exchange rate dynamics; see, for instance, Boswijk et al. (2007), Brock and Hommes (1997), Chiarella and He (2002), De Grauwe et al. (1993), De Grauwe and Grimaldi (2005a,b; 2006), Hommes (2010), Lux (1998), Westerhoff and Reitz (2003).

2.2 Dynamic forecasting rules

Despite the success in reproducing many of the stylized facts in exchange rate dynamics, the above set of models has been criticized. One important criticism concerns the (implicit or explicit) absence of learning dynamics. Specifically, in the De Grauwe and Grimaldi (2006) model, as in most of the behavioral finance literature, a fixed and time-invariant set of forecasting rules is assumed. That is, agents always choose (based on profitability, (see eqs. (8) and (9)) between the same *pre-determined* set of forecasting rules. Moreover, the parameters of the specific forecasting rules do not change over time. Put differently, agents do not learn (and/or adjust parameters) about the predictive power of the specific set of chartist or fundamentalist rules and, therefore, do not adjust their rules accordingly. Instead, they use identical rules across time.

We extend the De Grauwe and Grimaldi (2006) model by introducing an evolutionary selection mechanism which allows agents to adapt their forecasting rule over time. Specifically, we introduce a two-level decision tree where agents 1) first choose the type of forecasting rule based on profitability of each rule (i.e. chartist or fundamentalist) and 2) second, within each set of forecasting rules (either chartist of fundamentalist), compare (in terms of profitability) the existing forecasting rule to a new, randomly selected one. Although the new rules are selected randomly within each class, one may expect dominant forecasting rules to eventually emerge endogenously as a function of "natural selection".

We formalize the above extension by introducing at each point in time new competing forecasting rules. The new rules are obtained as a random permutation of existing forecasting rules by randomly changing the extrapolation parameter (for chartist rules) or the mean reversion (for fundamentalist rules), i.e. β and ψ, respectively. For the chartist forecasting rules, the degree of extrapolation is changed from the previous period value (β_{t-1}) to a new one equal to

$$\beta_t' = \beta_{t-1} + \varepsilon_{\zeta,t}, \qquad \varepsilon_{\zeta,t} \sim N(0, \sigma_{\varepsilon_\zeta}^2). \tag{10}$$

For the fundamentalist forecasting rules, this means that the new rule replaces the previous speed of adjustment (ψ_{t-1}) by a new one given by

$$\psi_t' = \psi_{t-1} + \varepsilon_{\psi,t}, \qquad \varepsilon_{\psi,t} \sim N(0, \sigma_{\varepsilon_\psi}^2). \tag{11}$$

As stated above, agents first decide on what type of rule to follow and then subsequently within each forecasting rule compare (by back-testing the respective rules) the profitability of the existing forecasting rule to a new (random) forecasting rule within the same class. If the profitability of the new rule is higher, agents shift to the new forecasting rule.

This additional decision layer introduces learning about the forecasting rules into the De Grauwe and Grimaldi (2006) model. Whereas the latter model assumes non-evolving forecasting rules, our extended model adapts the forecasting rules according to profitability. As a consequence, both the degree of mean reversion (for the fundamentalist rule) and the extrapolative parameter (for the chartist rule) eventually become endogenously determined. This endogenous process of selecting the fittest rules within each class of forecasting functions introduces some degree of learning and evolutionary rationality in the model. The crucial question, addressed in the next section, is whether or not behavioral models featuring complex dynamics are robust to this type of learning.

3. NUMERICAL ANALYSIS

In this section, we present the results of a numerical simulation of the model. We first illustrate the dynamics of the exchange rate keeping the degree of extrapolation (β) and speed of adjustment (ψ) constants over time. In other words, chartists and fundamentalists are allowed to switch from one group to the other, according to profitability, but are not allowed to revise their forecasting rules over time. These dynamics conform to the set up presented in De Grauwe and Grimaldi (2006). Subsequently, we show the effect on the exchange rate dynamics and on the expectations of agents when they can revise their forecasting rules over time. We focus on the set of forecasting rules emerging endogenously through the trial and error strategies explained in the previous section. Finally, we show those same results when we also allow the fundamental exchange rate to vary over time.

In all cases, we follow De Grauwe and Grimaldi (2006) and set the domestic and foreign interest rates to $r = r^* = 0$, the transaction cost to $C = 5$, the excess net market supply of foreign holdings $X_t = 0$, the number of lags in the chartist rule to $T = 5$, with $\alpha_1 = 0.44$, $\alpha_2 = 0.26$, $\alpha_3 = 0.16$, $\alpha_4 = 0.09$, and $\alpha_5 = 0.05$, and the level of risk aversion of chartists and fundamentalists to $\mu_c = \mu_f = 1$.

3.1 STATIC RULES AND CONSTANT FUNDAMENTAL RATE

We start by analyzing the dynamics of the exchange rate under static forecasting rules for both chartists and fundamentalists, keeping the level of the fundamental exchange rate constant ($S_t^* = 0$). The results were obtained simulating the model over 12,000 periods, following an initial shock ($S_0 = 2$) at time $t = 0$.

Figure 1 displays some of the dynamic properties of the model. The top left panel plots the variance of the exchange rate computed using the last 1000 periods of the simulation. The flat region in this figure shows the set of parameters β and ψ for which the exchange rate converges to its fundamental level. The area with positive variances shows the range of parameters for which the exchange rate never converges to a fixed point. This positive variance is hence an indicator of complex (non-linear) dynamics. In line with the literature and with De Grauwe and Grimaldi (2006) in particular, we find that complex non-linear exchange rate dynamics arise when the extrapolative force of the chartist forecasting rules (the β parameter) is sufficiently strong relative to the mean reverting fundamentalist forecasting rule (ψ); i.e. non-converging exchange rate dynamics typically arise for higher values of β and smaller values of ψ. The top right panel of Figure 1 shows in a two-dimensional space the same unstable region illustrated in the top left panel. The bottom left panel illustrates the attractor of the exchange rate in phase space for a specific set of parameters inside the instability region ($\psi = 0.14$ and $\beta = 0.86$), indicated in the top right panel of Figure 1. The attractor displayed in this figure suggests the presence of chaotic (or at least complex) exchange rate dynamics. Complex exchange rate dynamics therefore occur in a standard set-up with constant forecasting rules and fundamental exchange rate.

The bottom right panel of Figure 1 displays the time evolution of the exchange rate for this simulation. It can be seen that the exchange rate displays strong and complex cycles due to the fact that forecasting rules are in the unstable region. Since the forecasting rules of both chartists and fundamentalists are kept constant over time the exchange rate never converges to a steady state.

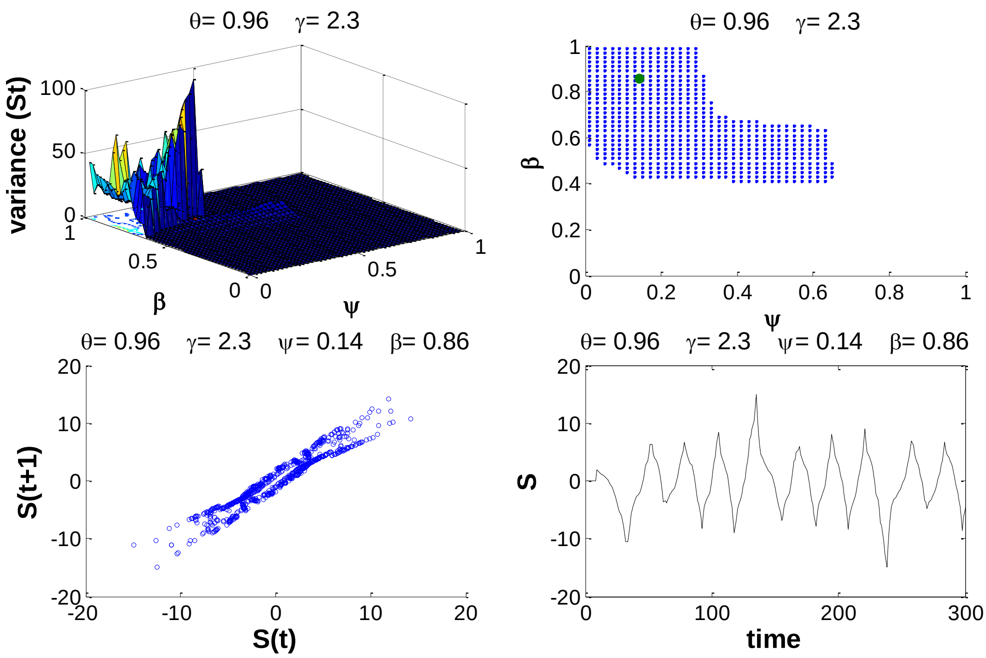

Figure 1 - Top left panel: Variance of last 1000 exchange rates. Simulation run of 12.000 periods; Top right panel: Mapping of the instability zone. The points show the values for the parameters β and ψ for which the exchange rate did not converge to its steady state value; Bottom left panel: Example of attractor obtained starting from a point inside the instability zone ($\beta = 0.86$ and $\psi = 0.14$).

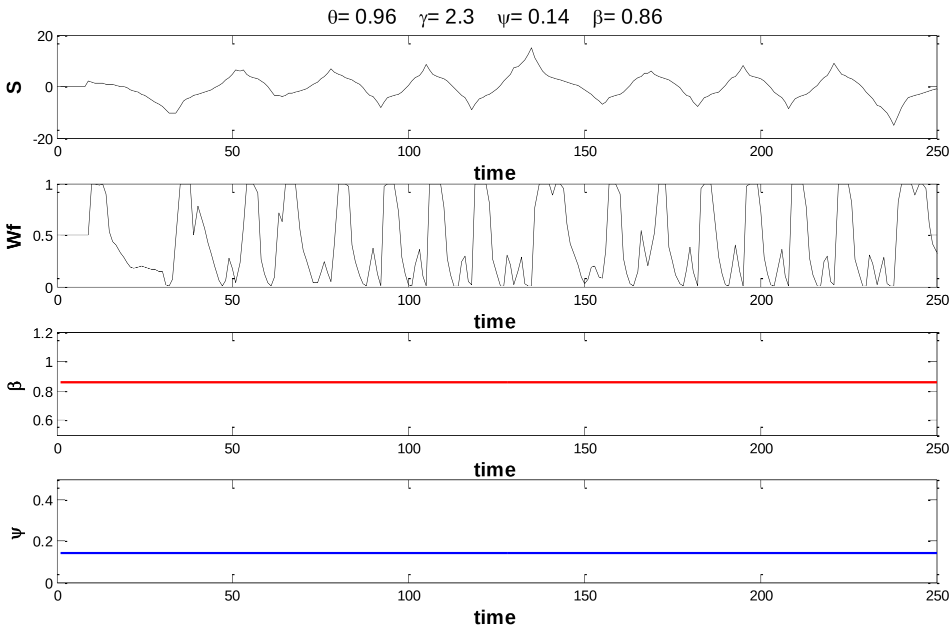

Figure 2 - First panel from the top: Exchange rate evolution for values for the parameters β and ψ inside the instability zone ($\beta = 0.86$ and $\psi = 0.14$). The exchange rate is initially set to 2; Second panel: Time evolution of the weight of fundamentalists in the market; Third panel: Value for the parameter β constant and equal to 0.86; Fourth panel: Value for the parameter ψ constant and equal to 0.14.

These dynamics are also shown in Figure 2 together with the evolution of the relative sizes of the group of chartists and fundamentalists. The bottom two panels of this figure show the fixed values for the parameters ψ and β. As can be observed, complex exchange rate dynamics are accompanied by cycles in the relative strength of chartists and fundamentalists.

Finally, note that the resulting exchange rate dynamics are highly stylized. More realistic dynamics are obtained by adding exogenous noise (e.g. liquidity traders or shocks to the fundamentals) to the model (see De Grauwe and Grimaldi (2006)). Moreover, this extended version of the model can be shown to replicate many of the observed empirical regularities in exchange rate dynamics: disconnect puzzles, volatility clustering, and fat tails.

3.2 Dynamic rules and constant fundamental rate

We now allow agents to reassess (learn by trial and error) their forecasting rules over time according to eqs. (10) and (11) while still keeping a constant fundamental exchange rate. Agents reassess (in terms of back-tested profitability) the existing forecasting rule (represented by β_t or ψ_t) against a randomly chosen alternative (represented by β_t' or ψ_t'). If the alternative rule outperforms the existing one the latter is replaced and a new set of forecasting rules is employed. As such, forecasting rules become dynamic and we study the set of steady state forecasting rules that endogenously arises in this model. Figure 3 displays the results of this exercise.

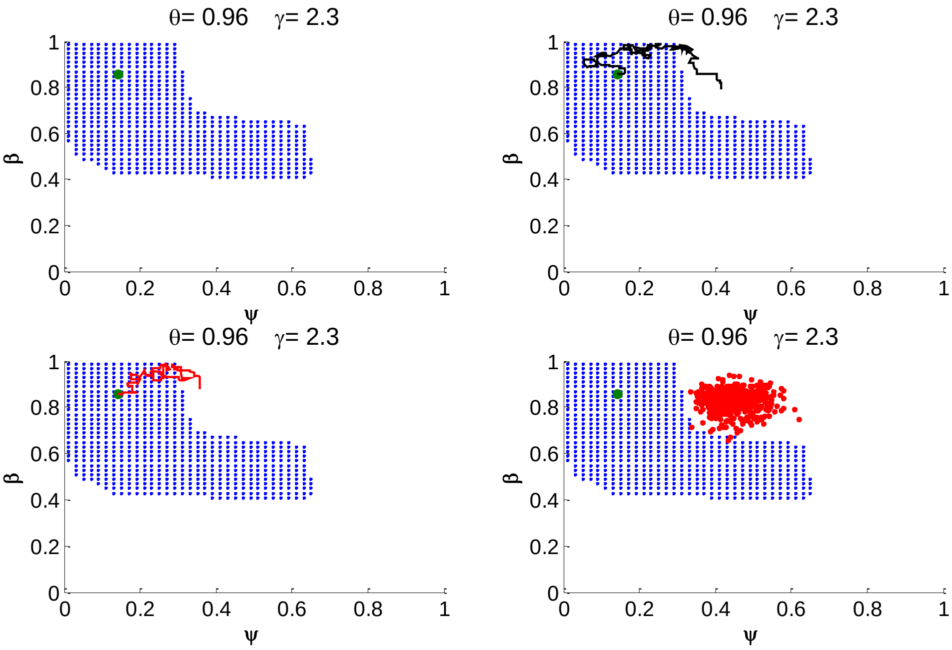

Figure 3 - Top left panel: Instability zone with the red dot showing the initial value for the parameters $\beta = 0.86$ and $\psi = 0.14$; Top right and bottom left panels: Examples of the time evolution of β and ψ allowing agents to adapt their forecasting rule over time. Bottom right panel: Final values of the parameters β and ψ for 500 simulations of the model.

The top left panel indicates the starting point of our simulation, which is inside the instability zone (same parameters as in the right panel of Figure 1). Without adding learning dynamics, the economy would remain in this point and would display the complex dynamics illustrated in the previous section. In this section, however, we allow for variation in the parameters of the forecasting rules (β_t and ψ_t) according to the decision rules explained above. As a result, forecasting rules *themselves* will become time-varying as old rules get replaced by new rules. The top right panel of Figure 3 illustrates the dynamics of the endogenously chosen forecasting rules for one simulation (12.000 periods). The solid line shows the time evolution of the parameters β and ψ as a consequence of the constant reassessment by agents of the most profitable rule. The result is clear. This learning process naturally leads agents to choose forecasting rules (parameter values for β and ψ) over time *outside* the instability region. In this simulation, therefore, complex (unstable) exchange rate dynamics are not robust to learning. Instead, dynamic adjustment of forecasting rules results in the convergence of the exchange rate to its fundamental value. For this simulation, convergence was reached after 252 periods.

The bottom left panel of Figure 3 illustrates the same process for a second simulation (with different sets of randomly selected forecasting rules) and the bottom right panel shows the final value for the parameters β and ψ for 500 simulations. The main conclusion emerging from this simulation exercise is that, for most simulations, the dynamic adjustment in forecasting rules leads to steady state forecasting rules implying stable exchange rate dynamics. Complex dynamics of the type proposed by De Grauwe and Grimaldi (2006) are therefore not always robust to simple (trial and error) learning rules. An intriguing feature of the model is however that some rules seem to converge to points within the unstable region.

Figure 4 shows the time evolution of the exchange rate, the weight of the fundamentalists, and the parameters β and ψ for the simulation shown on the top right panel of Figure 3. As in the case shown before (Figure 2), initially the exchange rate displays complex cycles. In this case, however, the initial forecasting rules do not seem optimal given the exchange rate dynamics and a random learning process is initiated. Specifically, learning through random selection seems to lead to new (better adapted) forecasting rules that imply higher mean reversion and somewhat lower extrapolation. As such, the weight of the fundamentalists relative to the chartists increases and stabilizes more the exchange rate, reinforcing the fundamentalist beliefs. Note that the learning process converges quickly once the parameters enter the stable region (this occurs around period 230 in this simulation). Once fundamentalists dominate sufficiently, they fully stabilize the market.

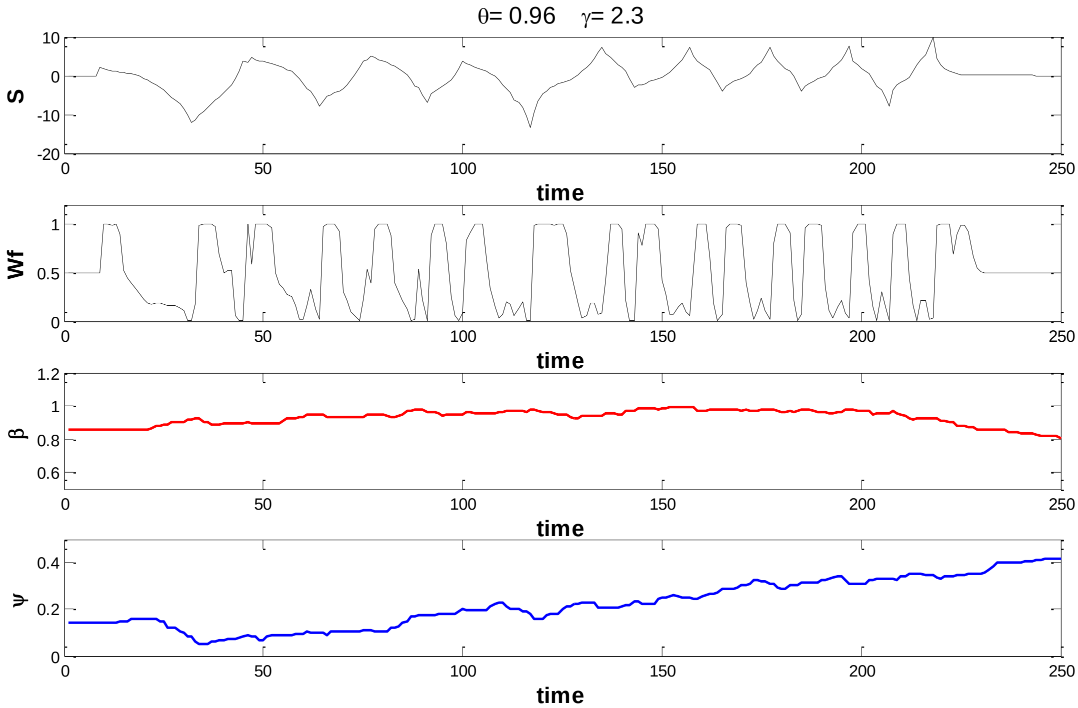

Figure 4 - First panel from the top: Exchange rate evolution allowing agents to adapt their forecasting rules over time. The parameters β and ψ are initially inside the instability zone ($\beta = 0.86$ and $\psi = 0.14$) and the exchange rate is initially set to 2; Second panel: Time evolution of the weight of fundamentalists in the market; Third panel: Time evolution of the parameter β; Fourth panel: Time evolution of the parameter ψ.

The previous results were obtained having as initial conditions the parameters β and ψ set to 0.86 and 0.14, respectively. Figure 5 shows the time evolution of these parameters for different starting values. The top left panel of this figure reproduces the results shown in the top right and bottom left panels of Figure 3. The other three panels display each two simulations starting from different points. Interestingly, the bottom left plot shows one simulation (blue line) in which although the set of parameters β and ψ are set initially inside the stable region, they cross the unstable region where the exchange rate converges to its steady state.

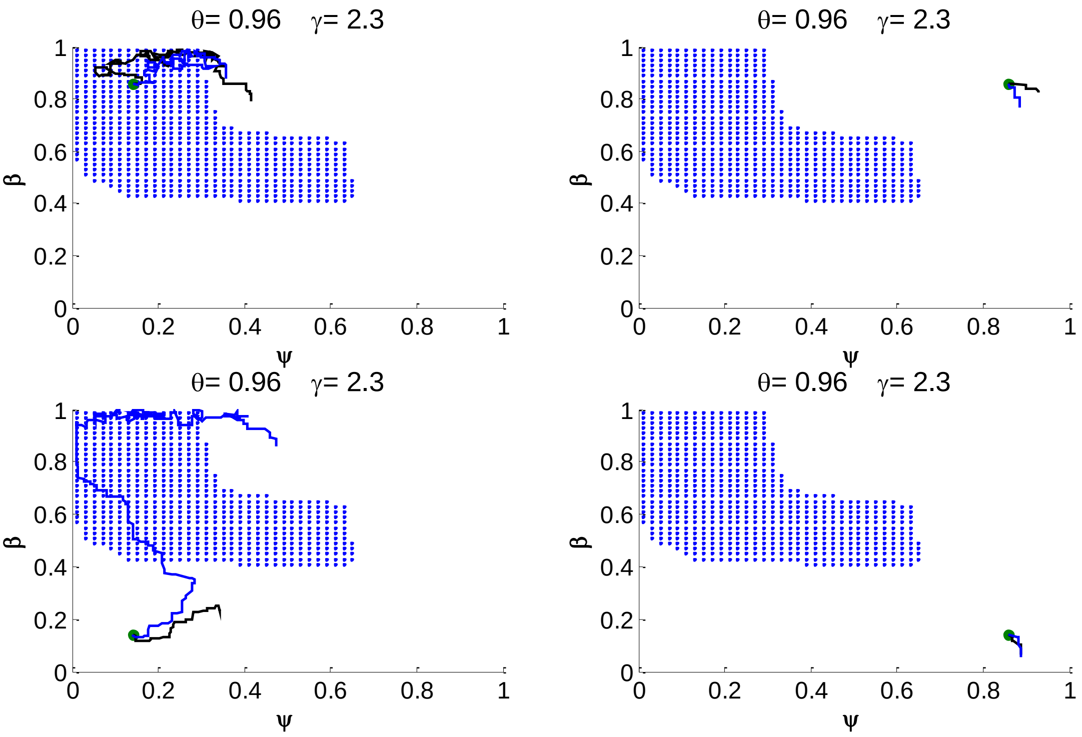

Figure 5 - Examples of the time evolution of the parameters β and ψ for different starting points of these parameters.

We generalize the procedure discussed before and simulate the model starting from a grid of initial values for the parameters β and ψ. This grid is illustrated in the left panel of Figure 6. The right panel of this figure shows the final values for these parameters once the exchange rates reaches its steady state. We observe that in almost all cases the steady state is obtained with the parameters β and ψ inside the stable region, indicating that in this stylized version not all complex dynamics seem to be robust to learning by trial and error.

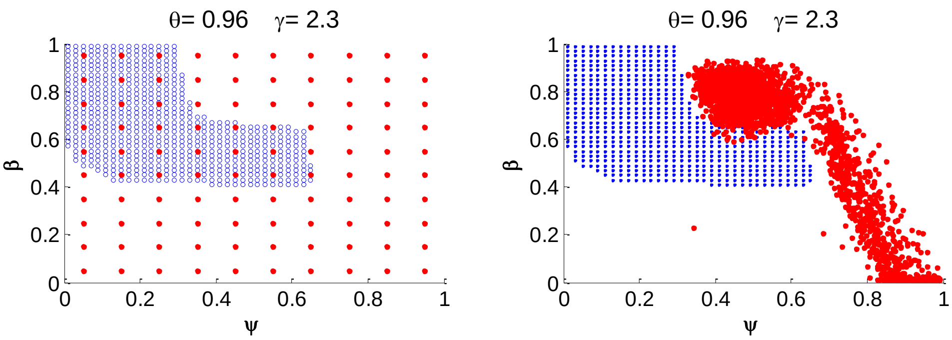

Figure 6 - Left panel: Instability zone with the red dots illustrating the initial values for the parameters β and ψ; Right panel: Final values of the parameters β and ψ starting from a grid of initial values for those parameters with a space equal to 0.02 between each point for both parameters.

3.3 Dynamic rules and time-varying fundamental rate

The above analysis shows that, in a highly stylized setting without shocks to the fundamental, learning reshapes and stabilizes the exchange rate dynamics. However, as forcefully argued in the behavioral finance literature one cannot simply abstract from shocks hitting the economy. In fact, heuristic rules of thumbs (i.e. chartist and fundamentalist forecasting rules) arise as a consequence of informational complexity, partly caused by exogenous unobserved shocks hitting the economy. In the same line of reasoning, De Grauwe and Grimaldi (2006) show that adding shocks to the nonlinear model is essential to replicate many of the stylized facts of exchange rate dynamics. The key question to answer then is to what extent nonlinear dynamics are robust to learning in models incorporating exogenous shocks.

In our final exercise, we therefore also allow the fundamental exchange rate to vary over time according to a standard random walk model, with a standard deviation equal to 0.1. Figure 7 presents the time evolution of the parameters β and ψ, which are set initially to 0.14 and 0.86, respectively, for one simulation of the model. We only plot the first 2000 periods.

A first important observation to make is that, in general, given shocks to fundamentals, learning rules no longer converge. Instead, random walk type of dynamics are observed for the forecasting rules, where over time forecasting rules (both chartist and fundamentalist) get replaced by alternatives. As the fundamental changes, typically other forecasting rules will be chosen as well and hence no convergence of forecasting rules arises. Moreover, and importantly, the region of complex dynamics (the shaded region in Figure 7) will be regularly visited by the model and gives rise to bursts of nonlinear and complex exchange rate dynamics. We therefore conclude that in a more extended model (including shocks), nonlinearities, i.e. bursts of complex exchange rate dynamics, will occur regularly in this model, despite the presence of learning dynamics. Finally, many of the nonlinear features of the basic model of De Grauwe and Grimaldi (2006) also carry over to this extended model. We briefly illustrate three of them: the exchange rate disconnect, volatility clustering, and fat tails. Each of these features is recovered in this model because the nonlinearities in the basic model of De Grauwe and Grimaldi (2006) remain relevant in the extended model (since the model will visit the complex dynamics region regularly).

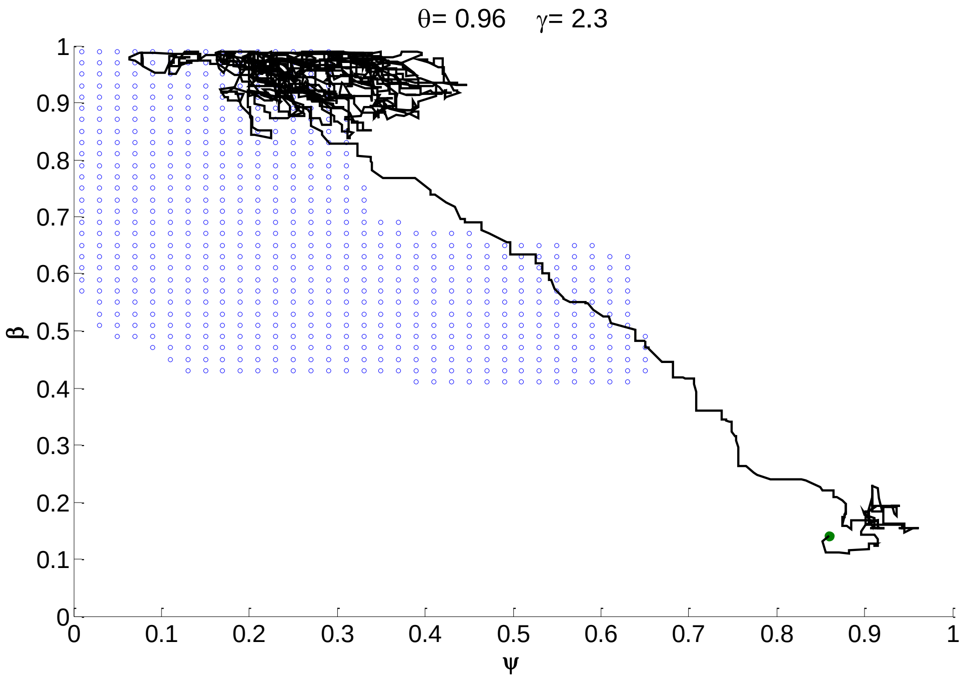

Figure 7 - First 2000 values for the parameters β and ψ allowing agents to adapt their forecasting rules and assuming a random walk process for the fundamental value of the exchange rate (standard deviation equal to 0.1). The initial values for the parameters are $\beta = 0.14$, $\psi = 0.86$ and S_0 =2.

First, the so-called exchange rate disconnect refers to the observation that exchange rate levels become disconnected from the underlying fundamental values for protracted periods of time. The current model contains this exchange rate disconnect feature as well. This can be seen clearly in Figure 8, which shows the evolution of the exchange rate (black solid line) and its fundamental value (blue bold line) for a simulation of the model.

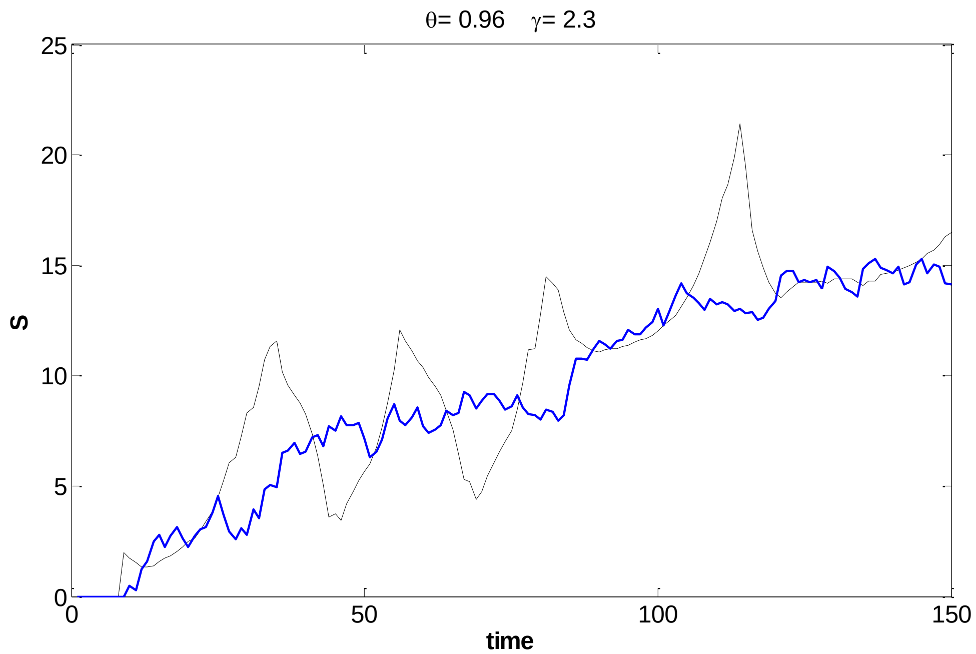

Figure 8 - Bold blue line: Fundamental value for the exchange rate following a random walk process with standard deviation equal to 0.1; Black line: Exchange rate allowing agents to adapt their forecasting rules over time. The initial values for the parameters are $\beta = 0.14$, $\psi = 0.86$ and S_0 =2.

Second, another empirical regularity is the fact that exchange rate volatility is clustered through time; periods of high (low) volatility in which the exchange rate exhibits large (small) changes tend to be clustered together. This feature is also observed in our model. Figure 9 exhibits the exchange rate return for a simulation run of 2000 periods. The presence of calm periods followed by turbulent ones is clear. This feature can also be seen examining the squared exchange rate return (Figure 10).

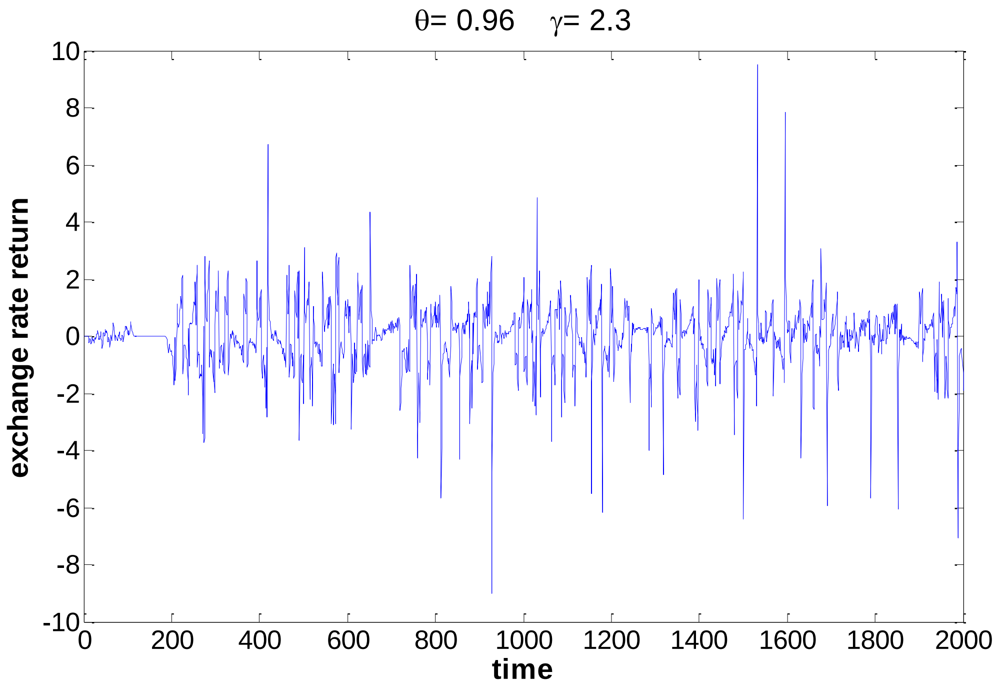

Figure 9 - Exchange rate return allowing agents to adapt their forecasting rules over time and assuming a random walk process with standard deviation equal to 0.1 for the fundamental value of the exchange rate. The initial values for the parameters are $\beta = 0.14$, $\psi = 0.86$ and S_0 =2.

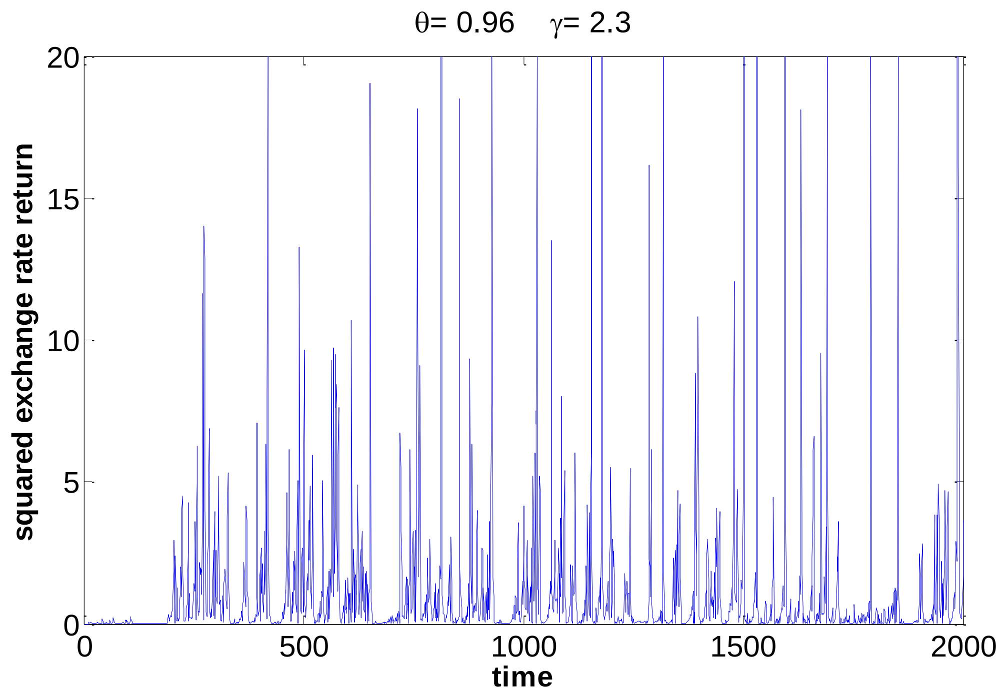

Figure 10 - Squared exchange rate return allowing agents to adapt their forecasting rules over time and assuming a random walk process with standard deviation equal to 0.1 for the fundamental value of the exchange rate. The initial values for the parameters are $\beta = 0.14$, $\psi = 0.86$ and S_0 =2.

Finally, the model replicates the stylized fact that exchange rates have so-called “fat tails”. As can be observed from Figure 10, extreme returns are observed relatively frequently, which indicates the presence of fat tails. More proper tests for fat tails report excess kurtosis. In line with De Grauwe and Grimaldi (2006), we find a level of kurtosis of the order of 12, which is indicative of the presence of fat tails.

Overall, we thus find that, in an environment with shocks, the findings of De Grauwe and Grimaldi (2006) carry over to models including learning dynamics.

Conclusion

In this paper, we presented a modified version of the heterogeneous agent model proposed by De Grauwe and Grimaldi (2006). The model allows for the presence of chartists and fundamentalists and takes into consideration transaction costs in the goods markets. We show that for a certain range of the parameters representing the degree of extrapolation of the chartist rule and the speed of adjustment implied by the fundamentalist rule, the exchange rate presents a complex dynamics and is governed by strange attractors.

We introduced an evolutionary selection mechanism allowing agents at each point in time to reassess the forecasting rule used by its group and to change it according to past profitability. With this mechanism in place, agents learn over time and naturally choose stable forecasting rules. The highly stylized version of the standard behavioral finance exchange rate model is therefore not robust to simple learning dynamics.

However, as argued in De Grauwe and Grimaldi (2006), exogenous shocks are an essential ingredient of behavioral finance models. Allowing for shocks, the learning dynamics no longer converge and the salient features of standard behavioral finance models (i.e. exchange rate disconnect, volatility clustering and fat tails) re-emerge, despite the introduction of a learning dynamics. Therefore, we conclude that behavioral finance models of the type developed by De Grauwe and Grimaldi (2006) not only offer essential new insights into the dynamics of foreign exchange rate markets but, moreover, are likely to be robust against the introduction of simple learning rules.

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